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Realization of Parking Task Based on Affine System Modeling

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Abstract

This paper presents a motion control system of an unmanned vehicle, where parallel parking task is realizedbased on a self-organizing affine system modeling and a quadratic programming based robust controller. Because of non-linearity of the vehicle system and complexity of the task to realize, control objective is not always realized with a single algorithm or control mode. This paper presents a hybrid model for parallel parking task in which seven modes for describing sub-tasks constitute an entire model.

Keywords: Parallel parking; Self-organizing affine system modeling; Quadratic programming

1. Introduction

Data-mining or data analysis is a new discipline lying at the intersection of statistics and artificial intelligence. Data-mining algorithm analyzes observational data sets to find each parameter's relationship with another, and summarizes the data in the way that is useful to the designer. GMDH (Group Method of Data Handling) is a well-known data mining technique that describes suspected dynamics in the form of minimal polynomials. Some of its applications are reported in the literatures: human driving skills were modeled by a polynomial expression in (Jong-Hae Kim et al., 2005) and the relationship between the size of a given input pattern and the base angular displacement of the planar space robot was well modeled in (Young Woo Kim et al., 2005). However, most of them are restricted to solve the conservative problems such as data interpretation, data clustering, pattern recognition etc. Control problems to bring out a desired data from the constructed model have been rarely found in the literatures.

This paper presents a motion control system of an unmanned vehicle, where parallel parking task is realized based on self-organizing affine system model and a quadratic programming based robust controller. Parallel parking task of the vehicle is highly constrained motion control problem, because of non-linearity of the vehicle system and complexity of the task to realize. Control objective is unlikely to be feasible with a single algorithm or control mode. In this paper, we present a new modeling method based on a hybrid dynamical system coupled with a self-organizing data-mining algorithm. Indispensable parts of both the dynamics and the task are separately constructed and seamlessly synthesized with a finite automaton model. We also present a new robust controller design method, where maximum modeling inaccuracy of the developed model is measured and control objective to track the reference value is robustly achieved under the designated size of the disturbances. In the proposed method, reproducibility of the developed model is guaranteed, as well as tracking performance of the state variables to their desired trajectories. We verify the usefulness of the proposed modeling and controlling methods.

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2. Affine system modeling for vehicular dynamics

This chapter presents an efficient modeling method for the vehicle dynamics operating at a low speed. When the vehicle is operated with a large steering angle applied, the directions of the front and the rear tires are different that without considering the lateral slips of each tire, high performance controller design is unlikely to be achieved for the exact motion control of the vehicle. Most of the models reported in the literatures ignored the lateral slips of each tire in case of large steering angle applied. For example, Ackermann's model which is one of most well-known models, assumed that $\sin(\beta)=\beta$, $\cos(\beta)=1$, and the cornering force is linear function of the slip as $F=\alpha(\beta)$. It is not suitable to use the models to the low speed operation control with possibly large steering angle applied as in parallel parking task.

The proposed algorithm extracts the relationship between the state variables (to be controlled) and other observational data sets, and compresses them in a polynomial affine system expression. Although conventional data mining algorithms such as Group Method of Data Handling, summarize the suspected dynamics, arbitrarily selecting the algebraic structure of the observational data sets, it is not still fully enough for direct application of the model.

From the perspective of the control system designer, it is convenient to observe how the system responds to the input. Especially when the model is developed in linear form to the input variables, the system can be easily controlled through the application of conventional control theories. The proposed algorithm below is a exploratory data mining procedure where the model to be developed, is skeletonized in a polynomial affine system form as follows.

2.1 Polynomial data mining algorithm For affine system modeling

Step 1 Initialization: Set: $\zeta_N=\phi$, $\zeta_L=\phi$, $\xi_N=\phi$, $\xi_N=\phi$, $\xi_L=\phi$, $l=0, \lambda=1$ and $r_{\min,0}=\infty$.

Step 2 Acquisition of Data Sets: Select the set of the target variables $\dot{\zeta}_N$ and its dimension Λ. List the observational data sets $\bar{\zeta}=\left[\bar{\zeta}_N\ \bar{\zeta}_L\right]$, and $\zeta_N\equiv\bar{\zeta}_N$, and $\zeta_L\equiv\bar{\zeta}_L$, where ζ_N denotes the set of the state variables to be nonlinearly evolved, and ζ_L denotes the set of the input variables to be linearly evolved, respectively. Each of the elements $\zeta_i(1\le i\le m)$ has n time series data. Note that $\zeta_i\ (1\le i\le m_N)\in\zeta_N$ and $\zeta_i(m_N+1\le i\le m)\in\zeta_L$. Subdivide the observational data sets into the training set S_T and the checking set S_C as follows,

$\zeta_{i,j}(\forall i, 1\le j\le n_t)\in S_T$ and $\zeta_{i,j}(\forall i, n_t+1\le j\le n)\in S_C$.

Step 3 Propagation of Variables: Combine all $\zeta_{i,j}\in S_T$ in the training set by the partial polynomials as follows.

if $\zeta_p\in\zeta_N$ and $\zeta_q\in\zeta_N$,

then $z_k=a_{k,0}+a_{k,1}\zeta_p+a_{k,2}\zeta_q+a_{k,3}\zeta_p^2+a_{k,4}\zeta_q^2+a_{k,5}\zeta_p\zeta_q$, $\xi_N\equiv\xi_N\cup\{z_k\}$

if $\zeta_p\in\zeta_L$ and $\zeta_q\in\zeta_L$,

then $z_k=a_{k,0}+a_{k,1}\zeta_p+a_{k,2}\zeta_q+a_{k,3}\zeta_q^2+a_{k,4}\zeta_p\zeta_q$, $\xi_L\equiv\xi_L\cup\{z_k\}$

if $\zeta_p\in\zeta_N$ and $\zeta_q\in\zeta_L$,

then $z_k=a_{k,0}+a_{k,1}\zeta_p+a_{k,2}\zeta_q+a_{k,3}\zeta_p^2+a_{k,4}\zeta_p\zeta_q$, $\xi_L\equiv\xi_L\cup\{z_k\}$

if $\zeta_p\in\zeta_L$ and $\zeta_p\in\zeta_L$,

then $z_k=a_{k,0}+a_{k,1}\zeta_p+a_{k,2}\zeta_q$, $\xi_L\equiv\xi_L\cup\{z_k\}$,

where $p,q\in\{1,2,\cdots,m\}$, $p\ne q$, $k\in\{1,2,\cdots,m(m-1)/2\}$.

Step 4 Computation of Partial Polynomial: Obtain the coefficients of the corresponding partial polynomial of Step 3 by applying the least square method that minimizes the difference between the target variable $\dot{\zeta}_\lambda$ and the dependent variable z_k as follows,

$$J=\sum_{j=1}^{nt}\left(\dot{\zeta}_{\lambda,j}-x_{k,j}\right)^2,$$

where n_t is the number of data in the training sets, and $z_{k,j}$ is the partial polynomial function of $\zeta_{p,j}$ and $\zeta_{q,j}$.

Step 5 Representativeness Assessment: The adequacy of each newly discovered variables z_k to represent the target variable is measured by following representativeness criterion.

$$r_k = \sqrt{\frac{\sum_{j=n_t+1}^{n}\left(\dot{\zeta}_{\lambda,j} - z_{k,j}\right)^2}{\sum_{j=n_t+1}^{n}\left(\dot{\zeta}_{\lambda,j}\right)^2}}$$

Note that $z_{k,j}$ is obviously in the checking set since $j \geq n_t$.

Step 6 Elimination of Least Effective Variables: Reorder z_k $\left(1 \leq k \leq m(m-1)/2\right)$ in the order of the size r_k from smallest to biggest. Screen out all z_k which do not satisfy $r_k < R_{th}$, where R_{th} is the threshold value. Do the following :

$$\zeta_N \equiv \zeta_N \cup \xi_N \quad \text{and} \quad \zeta_L \equiv \zeta_L \cup \xi_L$$

Step 7 Model Optimality Test: If $r_{\min,l} < r_{\min,l-1}$, set $l = l+1$, and go to Step 3, if $\lambda = \Lambda$, terminate with end otherwise set $\lambda = \lambda + 1, r_{\min,0} = \infty$, $l = 0, \zeta_N \equiv \overline{\zeta}_N, \zeta_L \equiv \overline{\zeta}_L$ and go to Step 3, where l is the iteration number, and $r_{\min,l}$ is the minimal number of at l th iteration.

The developed model can be represented as affine system form as follows

$$\dot{\zeta}_N = f(\zeta_N) + \sum g(\zeta_N)\zeta_L, \tag{1}$$

where f and g are the high-order polynomials expanded by the exploratory data mining. In this paper, $\frac{d}{dt}\left(\frac{1}{r}\right)$ and $\frac{d}{dt}(\theta)$ are selected as the target variables, where r is the radius of whirling motion and θ is the yawing angle. Also as the control inputs, the angular speed of the steering angle and the speed of the vehicle are used. The driving conditions used in this simulation are as follows: the vehicle is 3800*mm* long and 1527 *kg* weigh with the distance from gravity center to front wheel and real wheel of 1014 *mm* and 1676 *mm* respectively. And the radii of front and rear wheel is 277.566 *mm* and 286.387 *mm* respectively, where roll, pitch and yaw inertias are 606.1 kgm^2 and 2741.9 kgm^2, respectively. Note that although the functions f and g are possibly become very long and very high order polynomials depending on the behavior of the target variable, this model is very useful formation to realize the associated control objectives of the system, as we shall see in the next section.

3. Finite automaton model of automatic parking task

In this chapter, the proposed modeling method is applied to realize parallel parking task of unmanned vehicle system. Although the proposed affine system model accurately reproduces the vehicular behavior, it is still insufficient for realizing parallel parking task. This is because the objective task is highly constrained that the entire task is often unfeasible with a

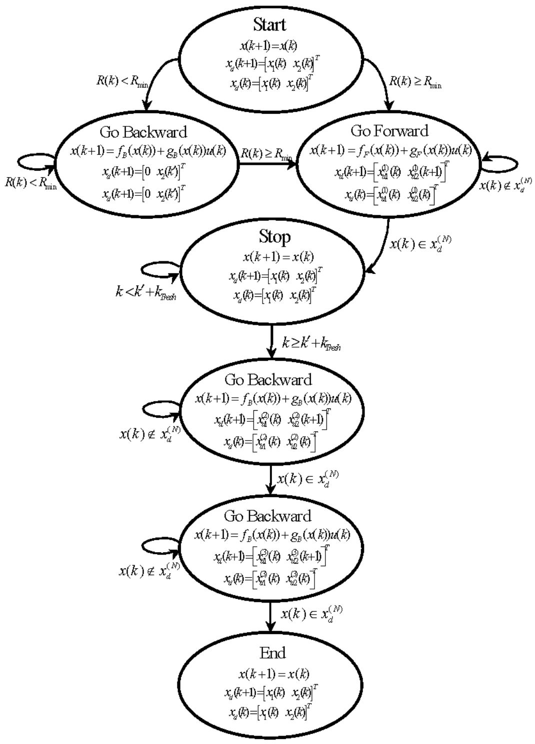

Fig. 1. Automaton Model of Automatic Parking Task.

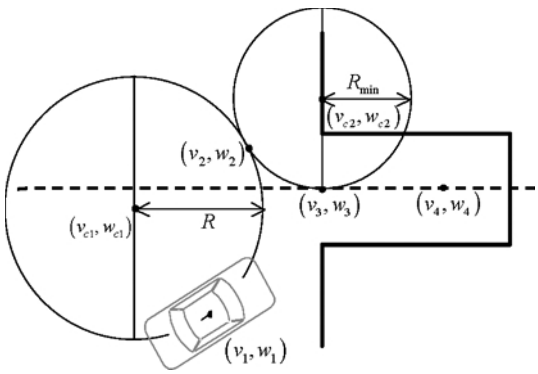

Fig. 2. Task Environment.

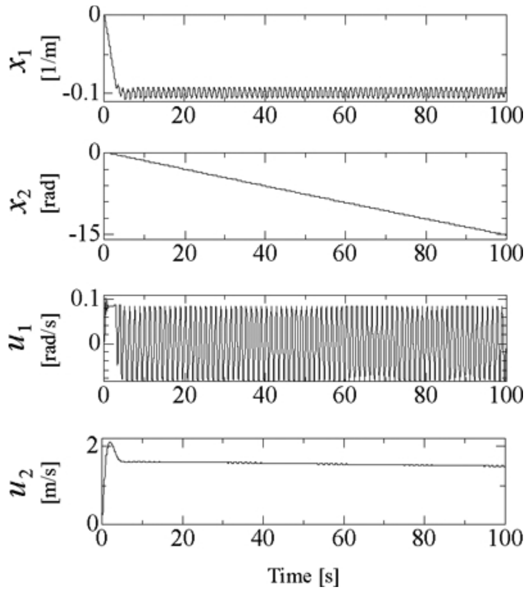

Fig. 3. Behavior of the variables.

single algorithm or control mode. Fig. 1 shows the automaton model of the automatic parking task, where some variables are defined as follows,

k' is the time instant when a new mode starts, $x_d(k)$ is the desired value of $x(k)$ and the subscript index (1), (2), and (3) indicate the corresponding positions of (v_2,w_2), (v_3,w_3) and (v_4,w_4) in Fig. 2 respectively, the subscript index (N) indicates the neighbor of the position and Rmin is the minimum radius we can realize the exact circular driving. The center (v_{c1},w_{c1}) and the radius from the current position are defined as follows,

$$v_{c1}=\tan^{-1}\theta\cdot(w_1-w_{c1})+v_1$$

$$w_{c1}=\frac{0.5\cdot(w_1-w_2)(w_1+w_2)-(v_1-v_2)(v_1\cdot\tan^{-1}\theta+0.5\cdot v_1-0.5\cdot v_2)}{(w_1-w_2)-(v_1-v_2)\cdot\tan^{-1}\theta}$$

$$R(k)=\sqrt{(v_1(k)-v_{c1})^2+(w_1(k)-w_{c1})}$$

In Fig. 2, the positions (v_{c2},w_{c2}), (v_3,w_3) and (v_4,w_4) are assumed to be easily acquired. The position (v_2,w_2) is easily obtained by drawing the tangential line as in the Fig. 2.

4. QP-based robust controller design

Although we developed a highly accurate model in the previous chapter, it still contains the modeling inaccuracy. It may come from the parametric uncertainty and/or the unmodeled dynamics. This chapter describes a new robust controller design method, where maximum modeling inaccuracy is considered. The Eq. (1) is represented in the discrete system form as follows

$$x(k+1)=f_D\left(x(k)\right)+\sum_i g_{D,i}\left(x(k)\right)u_i(k), \tag{2}$$

where $x \in R^2$ is the state vector, $u\in R^2$ is the control input vector. In order to make all state variables track the desired trajectories, a new variable is defined as follows,

$$s(k)=x(k)-x_d(k). \tag{3}$$

In order to guarantee the stability, the candidate Lyapunov function is given in terms of s(k) as follows,

$$V(k)=\frac{1}{2}\sum_{i=1}^{2}s_i(k). \tag{4}$$

The system is stabilized by applying the control input u (k) which satisfies for all i ∈ {1,2}, $s_i(k)\{s_i(k+1)-s_i(k)\}\le 0$, where $s(k+1)=\left[x_1(k+1)-x_{1d}(k+1)\ x_2(k+1)-x_{2d}(k+1)\right]^T$. It is obviously true that the above inequality satisfies the Lyapunov stability condition $\dot{V}\le 0$. Since the system's dynamics on the variable s is given as $\dot{s}(k)=0$ as t $\to\infty$, the corresponding control inputs are represented as follows,

$$\begin{bmatrix}\hat{u}_1(k)\\ \hat{u}_2(k)\end{bmatrix}=\begin{bmatrix}g_{D11} & g_{D12}\\ g_{D21} & g_{D22}\end{bmatrix}^{-1}\cdot\begin{bmatrix}-f_{D1}(x(k))+x_{1d}(k+1)+x_1(k)-x_{1d}(k)\\ -f_{D2}(x(k))+x_{2d}(k+1)+x_2(k)-x_{2d}(k)\end{bmatrix}. \tag{5}$$

The modeling inaccuracy on f is assumed to be bounded as follows,

$$\left|f_{D1}(x(k))-\tilde{f}_{D1}(x(k))\right|<F_1\ \left|f_{D2}(x(k))-\tilde{f}_{D2}(x(k))\right|<F_2,$$

where $\tilde{f}_D$ is the perfect dynamics i.e. it contains no modeling inaccuracy. The input variables are redefined in order to consider the modeling inaccuracy as follows,

$$\begin{bmatrix}\overline{u}_1(k)\\ \overline{u}_2(k)\end{bmatrix}=\begin{bmatrix}\hat{u}_1(k)-K_1(k)\mathrm{sgn}(s_1(k))\\ \hat{u}_2(k)-K_2(k)\mathrm{sgn}(s_2(k))\end{bmatrix}. \tag{6}$$

By using (6), the stability condition is modified as follows,

$$s_1(k)\left\{f_{D1}(x(k))-\tilde{f}_{D1}(x(k))-s_1(k)+g_{D11}(x(k))\cdot(-K_1(k)\mathrm{sgn}(s_1(k)))\right.$$

$$+g_{D12}(x(k))\cdot\left(-K_2(k)\mathrm{sgn}(s_2(k))\right)\Big\} < -\eta_1(k)\left|s_1(k)\right| \qquad (7)$$

$$s_2(k)\Big\{f_{D2}(x(k)) - \tilde{f}_{D2}(x(k)) - s_2(k) + g_{D21}(x(k))\cdot\left(-K_1(k)\mathrm{sgn}(s_1(k))\right)$$
$$+g_{D22}(x(k))\cdot\left(-K_2(k)\mathrm{sgn}(s_2(k))\right)\Big\} < -\eta_2(k)\left|s_2(k)\right|, \qquad (8)$$

where the elements of $\eta(k)=\left[\eta_1(k)\quad \eta_2(k)\right]^T$ take positive values. The Eqs. (7) and (8) contain the modeling inaccuracy, where K_1(k) and K_2(k) are the variable transformations of the input vector u(k), and η_i (k) is the variable which contributes to the convergence of the state variable x_i(k) toward x_{id}(k). These equations are transformed for application of the quadratic programming as follows:

i) in the case of $s_1(k) \geq 0 \wedge s_2(k) \geq 0$ or the case of $s_1(k) < 0 \wedge s_2(k) < 0$,

$$\begin{bmatrix} -g_{D11}(x(k)) & -g_{D12}(x(k)) & 1 & 0 \\ -g_{D21}(x(k)) & -g_{D22}(x(k)) & 0 & 1 \end{bmatrix} \begin{bmatrix} K_1(k) \\ K_2(k) \\ \eta_1(k) \\ \eta_2(k) \end{bmatrix} \leq \begin{bmatrix} -F_1 + \mathrm{sgn}(s_1(k))s_1(k) \\ -F_2 + \mathrm{sgn}(s_2(k))s_2(k) \end{bmatrix}$$

ii) in the case of $s_1(k) \geq 0 \wedge s_2(k) < 0$ or the case of $s_1(k) < 0 \wedge s_2(k) \geq 0$,

$$\begin{bmatrix} -g_{D11}(x(k)) & g_{D12}(x(k)) & 1 & 0 \\ g_{D21}(x(k)) & -g_{D22}(x(k)) & 0 & 1 \end{bmatrix} \begin{bmatrix} K_1(k) \\ K_2(k) \\ \eta_1(k) \\ \eta_2(k) \end{bmatrix} \leq \begin{bmatrix} -F_1 + s_1(k) \\ -F_2 - s_2(k) \end{bmatrix},$$

where the decision variables of the quadratic programming problem are $K_1(k)$, $K_2(k)$, $\eta_1(k)$ and $\eta_2(k)$. The control problem for the motion control of the unmanned vehicle system can be stated as follows,

Find $\overline{u}_1(k), \overline{u}_2(k), \eta_1(k), \eta_2(k)$ which minimizes the following performance criteria :

$$J = p_1\left(\overline{u}_1(k) - \overline{u}_{1,d}(k)\right)^2 + p_2\left(\overline{u}_2(k) - \overline{u}_{2,d}(k)\right)^2$$
$$+p_3\left(\eta_1(k) - \eta_{1,d}(k)\right)^2 + p_4\left(\eta_2(k) - \eta_{2,d}(k)\right)^2$$

subject to $u_{1,\min} \leq \overline{u}_1(k) \leq u_{1,\max}$ $\quad u_{2,\min} \leq \overline{u}_2(k) \leq u_{2,\max}$

$$u_1(k-1) + T_s\dot{u}_{1,\min} \leq \overline{u}_1(k) \leq u_1(k-1) + T_s\dot{u}_{1,\max}$$
$$u_2(k-1) + T_s\dot{u}_{2,\min} \leq \overline{u}_2(k) \leq u_2(k-1) + T_s\dot{u}_{2,\max}$$
$$-\eta_1(k) \leq 0 \qquad -\eta_2(k) \leq 0,$$

where $\overline{u}_{1,d}(k), \overline{u}_{2,d}(k), \eta_{1,d}(k), \eta_{2,d}(k)$ are the desired values for $\overline{u}_1(k), \overline{u}_2(k), \eta_1(k), \eta_2(k)$, and p_i, i ∈ {1,2, 3,4} is the positive weighting parameter. The objecttive function is then transformed with respect to each combination of s_1(k) and s_2(k).

5. Simulation results

In this chapter, we show some results to verify the usefulness of the proposed method. Fig. 3 shows the behavior of the state and input variables where the desired trajectory is set to have a circling movement with a radius r = 10 [m] and a given center. The parameters used in this simulation are as follows: the parameters for describing the desired trajectory, (v_1,w_1) =(0[m],0[m]), (v_{c1},w_{c1}) =(0[m],-10[m]), x_{1d}(k)=1/ 10[1/m], x_{2d}(k)= u_2 (k)/ 10 + x_2(k-1), u_{1d} (k)= u_1 (k-1), and u_{2d} (k)= u_2 (k-1), for obtaining performance optimization, p_1=p_2=1, p_3=p_4=10 and for defining the hard and soft constraints, $u_{1,max}$=13.56928 [deg/s] , $u_{2,max}$=1.79 [m/s], $u_{1,min}$= -9.876 [deg/s] , $u_{2,min}$=0.82 [m/s], $u_{1,max}$ =105.81096 [deg/s^2], $u_{1,min}$ = -79.81312 [deg/s^2], $u_{2,max}$ =0.10479926 [m/s^2], $u_{1,min}$=-0.10495454 [m/s^2]. We find that the vehicle follows well the desired trajectory that describes a circle with the radius of 10 meters. Because of the paper limitation, other data we obtained will be shown in the presentation.

6. Conclusions

This paper has developed a automatic parallel parking system of the unmanned vehicle operating at a low speed, based on the hybrid dynamical system coupled with self-organizing polynomial data-mining algorithm and the quadratic programming based robust controller. We firstly proposed a new modeling method, which summarizes non-linear dynamics in a polynomial affine formation with high granularity. We also proposed an efficient controller design method for the system for the developed statistically reproduced model. The proposed method has achieved the high tracking performance and the reliable reproducibility of the stochastic system. We have confirmed the usefulness of the proposed modeling and controlling method through some simulation results. Lastly we notify that the method proposed in this paper, are partly protected by our patent.

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